CHEM/CHPH/PHYS 703: Introduction to Nonequilibrium Statistical Physics

PROBLEM SET # 5, due at 5pm, Thursday, May 7, 2020

Problem 1

Eq. 28 of the paper "Feynman's ratchet and pawl: an exactly solvable model" (posted to the course website) shows a transition rate matrix whose elements are related to the physical parameters of the model (see Eqs. 5 and 27).

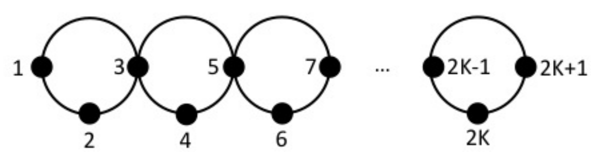
(a) Draw a six-state network that corresponds to this transition matrix, labeling the states as in the paper. Construct a maximal tree and identify the associated fundamental cycles.

(b) Compute the (dimensionless) affinity for each of these fundamental cycles:

$$A_c = \sum_{k=1}^{M} \ln \frac{R_{i_{k+1},i_k}}{R_{i_k,i_{k+1}}} \quad . \tag{1}$$

For each fundamental cycle whose affinity is not zero, briefly explain the net change that occurs in the two reservoirs when the system evolves once around the cycle.

Problem 2



A connected network with $N = 2K + 1$ states is depicted above, with nodes shown as large black dots, and edges as arcs. For $1 \leq k \leq K$, we have the following transition rates:

$$\begin{aligned} r(2k-1 \to 2k) &= d \\ r(2k \to 2k-1) &= u \\ r(2k-1 \to 2k+1) &= f \\ r(2k+1 \to 2k) &= b \end{aligned} \tag{2}$$

where $r(i \to j) = R_{ji}$ is the transition rate from i to j. All other transition rates are zero, hence some of the edges are directed.

Under these dynamics, the system evolves to a stationary distribution $\vec{\pi}$. Solve for π_{2K+1}/π_2.

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Problem 3

An overdamped Brownian particle at temperature β^{-1}, evolving in a harmonic potential that translates at a constant speed u, is modeled by the Langevin equation

$$\dot{x} = -\frac{k}{\gamma}(x - ut) + \xi(t) \tag{3}$$

where the noise term satisfies $\langle \xi(t)\xi(t') \rangle = 2D\delta(t' - t)$, and $\gamma^{-1} = \beta D$. Let $w(t)$ denote the work performed on the particle from time 0 up to time t, which satisfies $\dot{w} = -ku(x - ut)$; and let $p(x, w, t)$ denote the joint probability distribution that the particle is located at position x, and the work has a value w, at time t.

(a) Assuming the particle is in thermal equilibrium at $t = 0$, show that the $p(x, w, t)$ is a two-dimensional Gaussian for all $t > 0$, and solve for its first and second cumulants.

(b) Solve for the probability distribution of work values, $p(w, t)$, and show that it obeys the work relation $\langle e^{-\beta w} \rangle = 1$ for all $t \geq 0$.